UNIVERSITY OF MONTANA

Supplement to Guide to Graduate Programs in Mathematical Sciences

MATHEMATICAL SCIENCES
The University of Montana

2023–2024 Academic Year

Contents

1 Academic Year 2023–2024 Calendar

Fall Semester	
August 21–23	Advising & Registration
August 23–25	MA Comprehensive & PhD Preliminary Exams
August 28	First Day of Classes
September 4	Holiday: Labor Day
November 10	Holiday: Veteran's Day
November 22–24	Thanksgiving Vacation
December 11–15	Final Examinations
September	Deadline for Summer Graduate Research Scholarship reports
Spring Semester	
January 10–12	MA Comprehensive & PhD Preliminary Exam offered
January 17	Holiday: Martin Luther King Jr. Day
January 18	First Day of Classes
February 5	Spring Graduation Applications for MA and PhD due
February 19	Holiday: Presidents' Day
February/March	Bertha Morton Fellowship and Scholarship nominations due
March 1	Summer Graduate Research Scholarship applications due
March 18–22	Spring Vacation
May 6–10	Final Examinations
May 11	Commencement
1st Monday in June	Summer Graduation Applications for MA and PhD due
June 23	Final deadline for completion of all requirements for spring graduation

2 Graduate Faculty

2.1 Research Groups

Algebra	Kelly McKinnie Nikolaus Vonessen	Combinatorics & Optimization	Mark Kayll Cory Palmer
Analysis	Elizabeth Gillaspy Karel Stroethoff	Mathematics Education	Fred Peck Matt Roscoe Bharath Sriraman Ke Wu
Applied Mathematics	John Bardsley Leonid Kalachev Emily Stone	Statistics	Jonathan Graham
Data Science	Javi Perez-Alvaro	Topology	Eric Chesebro

2.2 Graduate Faculty

Name	**Degree, University**	**Research interests**
John Bardsley	Ph.D., Montana State University	numerical analysis
Eric Chesebro	Ph.D., University of Texas at Austin	geometric topology
Elizabeth Gillaspy	Ph.D., Dartmouth College	Noncommutative Geometry, C*-algebra
Jonathan Graham	Ph.D., North Carolina State University	statistics, spatial statistics
Leonid Kalachev	Ph.D., Moscow State University	asymptotic methods, math biology
Mark Kayll	Ph.D., Rutgers University	discrete mathematics
Kelly McKinnie	Ph.D., University of Texas at Austin	finite dimensional division algebras, the Brauer group, valuation theory & algebraic geometry
Cory Palmer	Ph.D., Central European University	graph colorings, extremal set systems & applied problems in graph theory
Fred Peck	Ph.D., University of Colorado	mathematics education
Javi Perez--Alvaro	Ph.D., Universidad Carlos III de Madrid	matrix analysis and numerical linear algebra
Matt Roscoe	Ph.D., University of Montana	mathematics education
Bharath Sriraman	Ph.D., Northern Illinois University	mathematics education
Emily Stone	Ph.D., Cornell University	dynamical systems, applied mathematics
Karel Stroethoff	Ph.D., Michigan State University	complex and functional analysis, operator theory
Nikolaus Vonessen	Ph.D., Massachusetts Institute of Technology	noncommutative rings and invariant theory, division algebra
Ke Wu	Ph.D., University of Minnesota	mathematics education

3 Communications

3.1 Mail room

The mail room is in room MA 101. Every graduate student has a mailbox, and it is important that this mailbox be checked periodically, since materials that may require timely responses will be distributed to graduate student mailboxes.

3.2 Email

In addition to written notices in mailboxes, official notices by email will be sent from time to time. It is your responsibility to maintain and check your official university email account. Once you've been assigned a Net ID, you have been assigned an email address through the university. You can find and manage your email addresses through the Personal Information tab in Cyberbear.

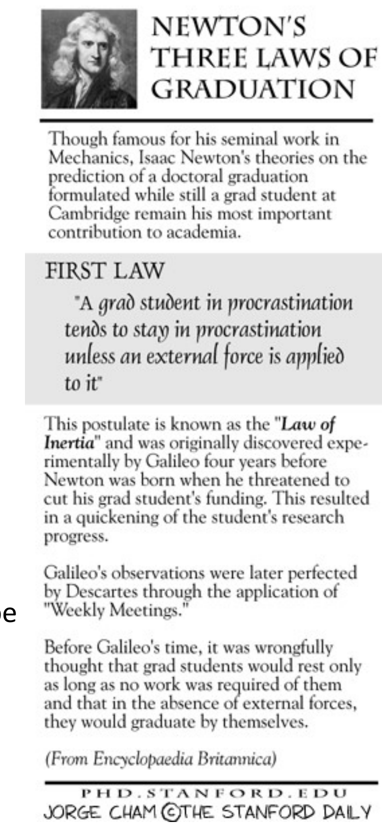
3.3 Graduate Committee

Current Graduate Committee:

Cory Palmer, Graduate Chair
Mark Kayll
Leonid Kalachev
Bharath Sriraman
Karel Stroethoff

Administrative assistance:

Linda Azure
Phone 243-5312
Email: linda.azure@umontana.edu



3.4 Webpage

Information for current students (including all graduate forms) can be found at the following web page:
https://www.umt.edu/math/graduate/current-grad.php.

4 Transfer Credits

4.1 Master's Degree Program

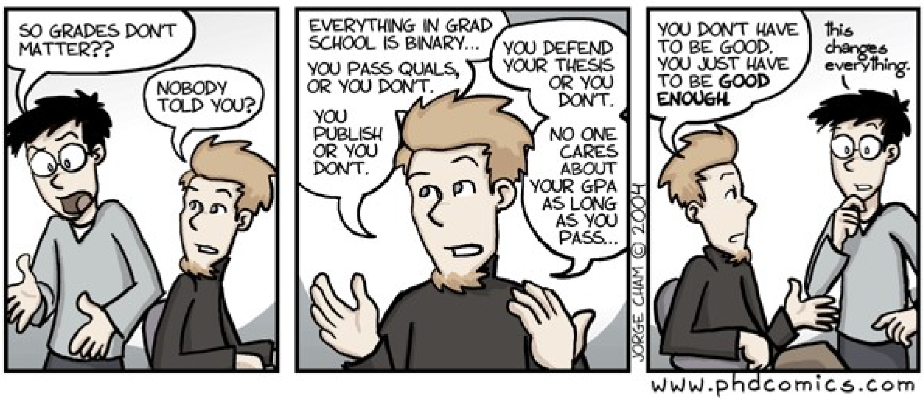
Students may transfer up to nine graduate/graduate non-degree semester credits or a full semester of graduate work on the recommendation of the program, after a semester of satisfactory work at UM. The 9 credit limit may be exceeded with recommendation from the program and approval from the grad school. The transfer credits must meet the following requirements:

1. The courses must have been taken for graduate credit. This information is verified by the Graduate School when the student submits a transcript of the transfer coursework.
2. Grades must be either an A or a B.

3. Credits must be earned at an institution that offers a graduate degree in the discipline of the course being transferred.
4. Credits must be applicable to the degree being sought.
5. The transfer request form is available on-line. Please complete the form and submit to the Graduate Chair.

4.2 PhD Degree Program

On the recommendation of the department and approval of the Graduate Dean, credits may be transferred (including an entire Master's Degree and/or credits from a Master's Degree program) from other institutions after a semester in residence.



Credits with grades other than A or B, thesis or correspondence credits, extension credits outside the Montana university system, or credits earned at institutions not offering graduate degrees in the discipline of the course are not transferable.

Graduate transfer credits are added to a student's record only if the student is in a graduate degree program and if the credit is applicable to the degree being sought.

Note. Since transfer credits must be applicable to the degree being sought, the Graduate Committee will only approve of transfer credits for courses offered through our graduate program. A request for transfer credits needs to include a listing of the courses at UM which are to be considered equivalent to the transfer credits. The transfer request form is available on-line.

5 Graduate Student Travel

5.1 Research & Creative Scholarship Fund

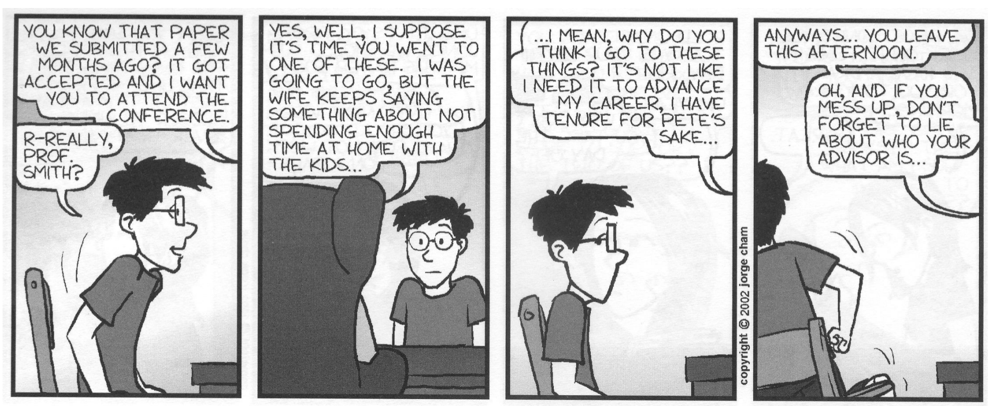
You can apply for travel funding and funding for other research and creative activities through the Associated Students of the University of Montana (ASUM)'s Research and Creative Scholarship Fund. Deadline is in the fall for travel in the spring/summer. See http://www.umt.edu/asum for full details.

The Graduate School allocates funds for graduate student travel (last academic year $10,000 was available for this purpose). Awards are available for graduate students intending to give talks/ papers at professional meetings. Graduate students can receive a travel award only once per academic year, and they can apply only for the period in which a paper is presented. Graduate students can apply for travel money if their papers are "pending acceptance" but must verify acceptance before the award is funded. Students are generally eligible to receive one travel award at the master's level and one at the doctoral level. Doctoral students who did not receive a travel award at the master's level are eligible for two awards at the doctoral level. Application needs to be made by the Graduate Program of the student's department. There will be several calls (last year there were three) to apply for these travel funds.

5.2 Departmental Graduate Student Travel Policy

Graduate Students in the Department of Mathematical Sciences may request travel funds for professional meetings. Requests will be considered by the Policy Committee and allocated on the following basis:

1. The Policy Committee will allocate funds and determine maximum reimbursement levels each year. Unused funds from fall semester will be made available for spring requests.
2. Up to the maximum level determined, the Department will provide $2 for each $1 the student contributes from personal and/or other sources. The departmental allocation will be more for students presenting papers, but students may request funding for participation.



In recent years maximums of $600 for students presenting a paper and $400 for participation if no presentation is given were followed. Requests for travel money should be submitted to the Graduate Program Chair prior to travel. While it is possible to receive funding for more than one trip in an academic year, preference will be giving to applicants requesting funding for the first trip. The form can be found on the math grad website.

6 Graduate Student Scholarships and Fellowships

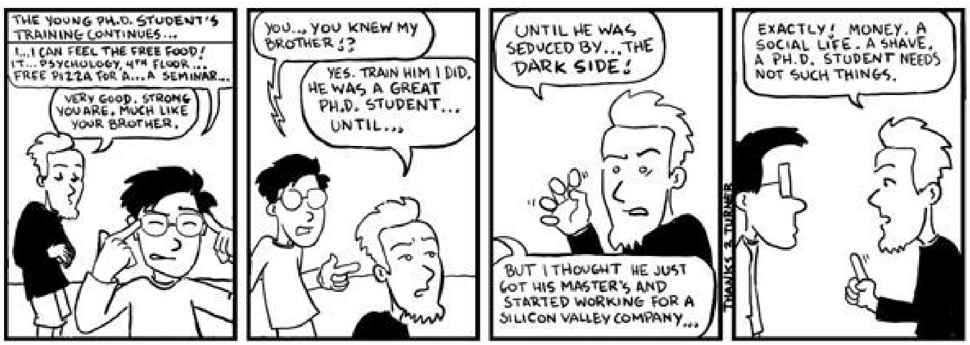
6.1 Graduate School Scholarships and Fellowships

Bertha Morton Fellowships and Scholarships

Award amounts are $2000 and $3000, respectively. Nomination documentation and recommendations should be based on the student's academic record and accomplishments in one or more of the following areas:

- Honors and awards
- Professional certifications and credentials
- Evidence of research and other academic achievements
- Evidence of professional and community achievements
- Additional achievements and creative activities

Our Graduate Program may nominate up to two graduate students for these awards. The Graduate Committee will select the students who will be nominated. Application for these awards is in February, you will get an announcement in January.



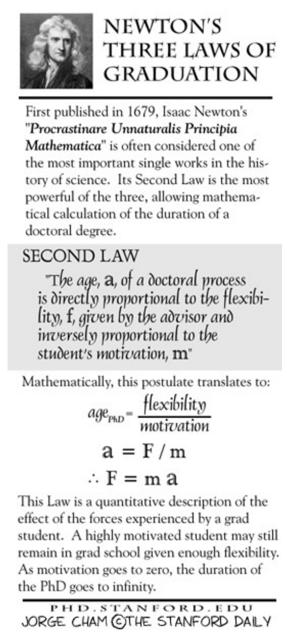
JORGE CHAM ©THE STANFORD DAILY

Andrew and Elizabeth Lassen Scholarship

A renewable $2,000 a year graduate student award.

Eligibility requirements:

- Must be a single parent with dependent children residing in the home, with financial need.
- Full time student.
- GPA of 3.25 or above.



Dennis and Phyllis Washington Scholarship

An annual $2,000 award.

Eligibility requirements:

- Have financial need.

More information on these and other scholarships and how to apply can be found on the graduate school's website:

https://www.umt.edu/grad/explore/tuition-and-funding/scholarships-and-fellowships.php

6.2 Departmental Scholarships

Summer Graduate Research Scholarships in Mathematical Sciences

Award amount: up to $3200 per award. Any graduate student in the Department of Mathematical

Sciences may apply. Preference will be given to graduate students in the Ph.D. program, especially students who have passed their preliminary exams, but strong applicants in the Master's program are also encouraged to apply. Applicants are requested to submit an application containing the following data: sketch of applicant's background (place of origin, schools attended, degrees, etc.), list of all courses and seminars taken at UM, including names of professors and grades, list of examinations taken (with dates), future academic plans (including courses, seminars and examinations the applicant is planning to take), a narrative detailing the applicant's research plans for the summer during which support is requested sponsored by a departmental faculty member, other pertinent information (publications, presentations, etc.). Application for these awards will be made in March. You will get a call for applications in the spring.

7 MA Comprehensive/PhD Preliminary Exams

The written MA Comprehensive/PhD Preliminary Examinations are closed-book exams in which students are not allowed to bring notes prepared in advance. However, the Examination Committee is allowed to include, as part of an exam, anything (for example, tables) that the committee deems necessary for students to have while taking the exam. The following pages contain descriptions of the MA Comprehensive/PhD Preliminary Examinations in each of the subject areas. Old exams are available in the Math Office.

Preliminary exam exemption

Students who completed their MA degree in the department before entering the PhD program can be exempted from a preliminary exam in the area of their MA emphasis if they performed exceptionally well on the MA comprehensive exam, receiving a pass at the PhD level.

7.1 Algebra

The key topics for the graduate exam in algebra are:

Prerequisites: Properties of integers (e.g., division algorithm, the greatest common divisor as a linear combination, primes), mathematical induction, equivalence relations, elementary properties of functions (e.g., injective, surjective, bijective, invertible).

Group Theory: Subgroups, cyclic groups, examples of groups (e.g., symmetric groups, dihedral groups, GL_n, SL_n), cosets, Lagrange's Theorem, normal subgroups, factor groups, group homomorphisms and isomorphisms, isomorphism theorems, external and internal direct products, Fundamental Theorem of Finite Abelian Groups.

Ring Theory: Ideals, factor rings, ring homomorphisms, isomorphism theorems, integral domains, the field of quotients of an integral domain, prime and maximal ideals, principal ideal domains, polynomial rings (with emphasis on the case where the coefficients lie in a field or in the ring of integers), factorization of polynomials, irreducibility tests (including Eisenstein's Criterion), Gauss' Lemma.

Linear Algebra and Vector Spaces: Linear independence, bases, dimension, matrices, solving systems of linear equations, linear transformations, the matrix of a linear transformation with respect to two bases, rank, trace, elementary properties of determinants, eigenvectors, eigenvalues, diagonalization.

Field Theory: Extension fields, roots of polynomials, adjunction of elements, splitting fields, algebraic and transcendental extensions, finite extensions, finite fields and their subfields.

Most of these topics are usually covered in the undergraduate algebra sequence Math 421/422, with the exception of some of the linear algebra topics, which you should have seen in your undergraduate linear algebra course. The exam does, however, require the maturity of reasoning expected from a graduate student. It is therefore strongly recommended to take at least one additional proof-oriented course (400-level or above, need not be in algebra) before taking the algebra exam. Taking the graduate algebra sequence is neither necessary nor required (although it certainly does help).

Some introductory books on Abstract Algebra:

- Joseph A. Gallian, *Contemporary Abstract Algebra*, fifth edition, Houghton Mifflin, 2002.
- N. Herstein, *Topics in Algebra*, second edition, John Wiley and Sons, 1975.
- Thomas W. Hungerford, *Abstract Algebra, An Introduction*, second edition, Saunders, 1997.

To prepare for the linear algebra part of the exam, first read the relevant section on vector spaces in a textbook on abstract algebra. Then review the material from your undergraduate linear algebra class. A good proof-oriented reference for linear algebra (which contains much more material than covered on this exam) is the book

- Stephen H. Friedberg, Arnold J. Insel, and Lawrence E. Spence, Linear Algebra, third edition, Prentice Hall, 1997.

7.2 Analysis

Here are some of the key topics to be covered by the exam. The students are expected to demonstrate: (1) The ability to solve problems, combine, relate and utilize various of these topics, as well as other topics covered by all 400-(and lower-) level Analysis (also Calculus, Abstract Mathematics) courses offered at UM; (2) An understanding of the breadth and a substantial knowledge of mathematics topics and their deep interconnecting principles, and of the interplay among problem solving, theory, and applications; (3) An awareness of the abstract nature of theoretical mathematics and the ability to write proofs.

Though the exam is based on 400-level courses in Analysis, taking any particular 400-level course is not necessary nor sufficient for proper preparation. The exam should not be viewed as a "final examination" for a specific course.

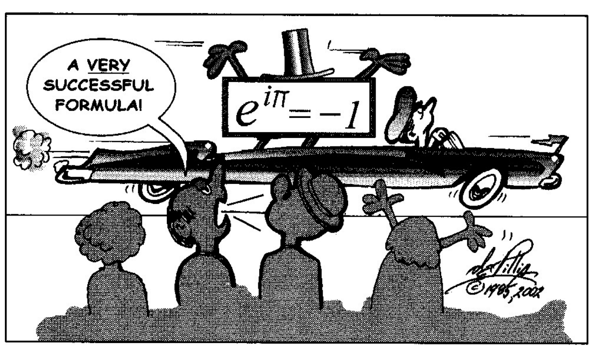
A short list of literature, providing the essential material is included, which may be helpful in studying the topics for the exam, though it is not expected that the students will study all of the listed books before taking the test. Looking at exams from the past will be a good idea.

Key Topics

1. The real number system. Properties and basic theorems.
2. Theory of metric spaces and functions on them. Properties and basic theorems (including Baire category theorem, Banach fixed point theorem etc.).
3. Uniform convergence of functions. Relationship with continuity, differentiability, integration. Properties and basic theorems (including Weierstrass, Arzela-Ascoli theorems etc.).
4. Lebesgue integration. Properties and basic theorems (including Lebesgue dominated and other convergence theorems etc.).
5. Analytic functions, complex integration.
6. Singularities and applications to contour integration.
7. Harmonic functions, spaces of analytic functions.

Recommended Reading

- R. P. Boas, *Invitation to Complex Analysis*, Random House, 1987.
- B. D. Craven, *Lebesgue Measure and Integral*, Pitman, London, 1982.
- W. R. Derrick, *Complex Analysis and Applications*, Brooks/Cole, 1983.
- L.-S. Hahn and B. Epstein, *Classical Complex Analysis*, Jones & Bartlett, Sudbury, Massachusetts, 1996.
- J. M. Howie, *Complex Analysis*, Springer, London, 2003.
- H. A. Priestley, *Introduction to Complex Analysis*, Oxford University Press, 1985.



- M. Rosenlicht, *Introduction to Analysis*, Dover, New York, 1986.
- W. Rudin, *Principles of Mathematical Analysis*, McGraw-Hill, New York, 1976.
- D. A. Sprecher, *Elements of Real Analysis*, Dover, New York, 1987.
- R. S. Strichartz, *The Way of Analysis*, Jones and Bartlett Publishers, Boston-London, 1995.
- K. R. Stromberg, *An Introduction to Classical Real Analysis*, Wadsworth, Belmont, 1981.

7.3 Applied Mathematics

Topics:

Ordinary Differential Equations: solution techniques for initial and boundary value problems, linear theory (equations and systems), stability, numerical methods, power series methods, Sturm-Liouville theory, Laplace transform and Fourier series methods, matrix methods.

Partial Differential Equations: Method of characteristics, Heat, Wave, and Laplace equations, separation of variables method and orthogonality, Poisson integrals, maximum/minimum principles, Laplace transforms, traveling waves, Green's functions.

Modeling: Linear and nonlinear difference and differential equations and systems, eigenvalues and eigenvectors, phase-planes, stability, and direction fields, elementary background in period doubling, bifurcation theory, and chaos.

Numerical Analysis: Basic techniques of solving ordinary and partial differential equations numerically, LU decomposition of matrices, QR and power methods of finding eigenvalues, the four linear spaces associated with a matrix (row and column space, kernel, cokernel), orthogonality of eigenspaces and numerical methods related to these concepts.

Texts:

1. Most of the topics mentioned under the headings of Ordinary Differential Equations, Partial Differential Equations, Linear Algebra, and Numerical Analysis can be found in *any* Advanced Engineering Mathematics text. For example:
    - William E. Boyce, Richard C. DiPrima, Elementary Differential Equations, J. Wiley 2013
    - John M. Davis, Introduction to Applied Partial Differential Equations, Freeman & Co., NY 2013
2. Most of the topics mentioned as modeling appear in the following four references:
    - L. Edelstein-Keshet, Mathematical Models in Biology, Random House, 1988.
    - Steven H. Strogatz, Nonlinear Dynamics and Chaos, Westview Press
    - M. Mangel & C. Clark, Dynamic Modeling in Behavioral Ecology, Princeton, N.J. 1988.
    - L. Perko, Differential Equations and Dynamical Systems, Springer-Verlag, 1991.
    - J. Guckenheimer & P. Holmes, Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields, Springer-Verlag, 1983.

7.4 Combinatorics & Optimization

Outline of Topics for the M.A. Comprehensive/Ph.D. Preliminary Exam

Combinatorics

1. enumeration (of the basic combinatorial objects: functions, permutations, subsets [combinations], multisets, set partitions [Stirling and Bell numbers], vector subspaces [Gaussian coefficients], compositions of integers, partitions of integers, etc.)
2. combinatorial theory (double-counting, recurrence relations, generating functions, pigeonhole principle, inclusion-exclusion, systems of distinct representatives, extremal set theory)
3. combinatorial structures (set systems [families of sets, hypergraphs], matroids, designs, finite geometries, partially ordered sets)

Graph Theory

1. graphs (trees, matchings, connectivity, Euler tours, Hamilton cycles, coloring, planarity, extremal graph theory, Ramsey theory, random graphs)
2. directed graphs and networks (tournaments, max-flow min-cut theorem, Menger's theorem, matrix-tree theorem)
3. algorithms (e.g.: searching, optimization [shortest paths, minimum spanning trees, maximum matching {bipartite and non-bipartite}], RNA sequencing)

Optimization

1. linear programming algorithms (geometric approach, simplex method, dual simplex method, primal-dual algorithm, network simplex method)
2. linear programming theory: duality (duality theorems, complementary slackness, sensitivity analysis, economic interpretation); geometry (convexity, hyperplanes, half spaces, polytopes, polyhedra)
3. integer programming (LP integrality theorems, transshipment, assignment, and transportation problems, network-flow problems)

Suggested Reading

- Graph *Theory*, Bondy and Murty, Springer 2008
- *Combinatorics: Topics, Techniques, Algorithms*, Cameron, Cambridge 1994
- *Linear Programming*, Chvátal, Freeman 1983
- *Graph Theory*, 4e, Diestel, Springer 2010
- *Discrete Mathematics with Graph Theory*, 3e, Goodaire & Parmenter, Prentice-Hall 2005
- *Linear Optimization: The Simple Workbook*, Hurlbert, Springer 2010
- *A Course in Combinatorics*, 2e, van Lint & Wilson, Cambridge 2001

- *Linear Programming and Its Applications*, Strayer, Springer-Verlag 1989
- *Introduction to Graph Theory*, 2e, West, Prentice-Hall 2001

Remarks: Students are expected to demonstrate proficiency beyond an introductory level. One purpose of the exam is to encourage students to synthesize material spanning several C & O courses. However, to pass the examination, it is not necessary (nor sufficient) to take specific courses. Likewise, there are many good texts that may be consulted in addition to—or in place of—those on the list above. For example, any text or other materials used in C & O courses fit this description. As a guiding principle, appropriately prepared students should expect to demonstrate insight and sophistication appropriate for graduate-level work.

7.5 Data Science

Numerical Analysis (M540)

Textbook: Fundamentals of Matrix Computations, by David Watkins.

Floating point arithmetic:

1. Floating Point Numbers
2. Rounding Errors and Cancelation

Linear Systems:

1. Cholesky Factorization
2. LU Factorization, Gaussian Elimination, and Gaussian Elimination with Partial Pivoting
3. Condition Numbers

Least squares problems:

1. Linear Regression
2. Normal Equations
3. QR Factorization, Gram-Schmidt, Reflectors and Rotators

Eigenvalues:

1. Power Method, Inverse Iteration, Shift-and-Invert Iteration
2. QR Algorithm. Implicit QR Algorithm
3. Singular Value Decomposition (SVD)

Krylov methods:

1. Krylov Subspace
2. Arnoldi Method (for solving large and sparse eigenvalue problems)

Advanced Data Analytics (M561)

Textbook: Algorithms for Data Science

Data Reduction and Information Extraction; Reduction of Massive Data Sets

Scalable Algorithms; Algorithms for Processing Massively Large Data Sets.

Hadoop/MapReduce; Distributed Computing.

Data Visualization

Building Predictors and Classifiers from first Principles

Data Analytics Theory (M562)

Models:

1. Cost Functions
2. Convex Functions
3. Gradient Descent Method.

Prediction Problems:

1. Linear Regression
2. The Mean Squared Error and its Gradient
3. Gradient Descent ant its Variants
4. Overfitting
5. Regularization: Ridge, Lasso and Early Stopping of Gradient Descent
6. Prediction Trees

Binary and multiclass classification:

1. Logistic and Softmax Regression
2. Naïve Bayes
3. K-Nearest Neighbors
4. Classification Trees

Cluster Analysis:

1. K-Means
2. Hierarchical Clustering
3. Spectral Clustering

Dimensionality Reduction:

1. Principal Component Analysis (PCA)
2. T-SNE

Python:

1. Dictionaries, lists and sets
2. List and Dictionary comprehension
3. Matplotlib and NumPy
4. Reading and saving files

7.6 Mathematics Education

Topics for this exam may be chosen from section (a) or sections (a) & (b).

(a) Mathematics Content. Students are expected to demonstrate proficiency in topics typically covered in 300–400 level courses such as Algebra/ Geometry, Probability & Statistics, Number Theory, Mathematical Modeling and History of Mathematics. Topics for potential exam questions include the following:

- Algebraic structure of the real numbers and subsets of that systems.
- Equivalence relations and equivalence classes.
- High school concepts extending from the ring of polynomials.
- Concepts in mathematics related to Euclidean constructions.
- Transformation groups of isometries.
- Logic of proofs.
- Transfinite numbers.
- Diophantine equations and their relation to geometric concepts.
- Congruences and modular arithmetic.
- Measures of central tendency and dispersion.
- Probability distributions.
- Mathematical modeling.
- Historical development of specific topics (for example, the limit).

Texts used recently

- Gallian, J. (2001). Contemporary Abstract Algebra. Houghton Mifflin Company.
- Kay, D. (2000). College Geometry: A discovery approach. Pearson Educational Publishers.
- Meyer, W. (1999). Geometry and its applications. Harcourt Brace & Company.
- Moore, D. (1999). Basic practice of Statistics. W.H. Freeman & Co.
- Burton, D. (2001). Elementary Number Theory. Houghton Mifflin Company.
- Giordano, F., Fox, W. & Weir, M. (1997). First course in mathematical modeling. Thomson Learning.

- Katz, V. (1998). A History of Mathematics. Addison Wesley.
- Burton, D. (1998). The History of Mathematics. McGraw Hill.

(b) Mathematics Pedagogy. Students are expected to demonstrate literacy on the theories of learning and teaching of mathematics as well as literature on curriculum, evaluation and philosophy of mathematics education. Students are also expected to have had experience reading and critically evaluating original research, and to have developed an awareness of and an appreciation for various qualitative and quantitative research methods in mathematics education.

Suggested Readings:

- Cobb, P., Goldin, G. et. al (1996). (Eds.), Theories of mathematical learning. Hillsdale, NJ: Lawrence Eribaum Associates.
- Ernest, P. (1991). The philosophy of mathematics education. Briston/ PA: The Falmer Press.
- Grouws, D.A. (Ed.). (1992). Handbook of research on mathematics teaching and learning. Simon and Simon Schuster, NY.
- Kelly, A. & Lesh, R. (Eds.) (2000). Research design in mathematics and science education. Mahwah: NJ: Lawrence Eribaum and Associates.
- Krutetskii, V. A. (1976). The psychology of mathematical abilities in school children. (J. Teller, trans. and J. Kilpatrick & I. Wirszup, Eds.). Chicago: University of Chicago Press.
- Skemp, R. (1986). The psychology of learning mathematics. Penguin Books.

Additional readings may include: a selection of articles from various math-ed journals, readings on the history of reform.

7.7 Statistics

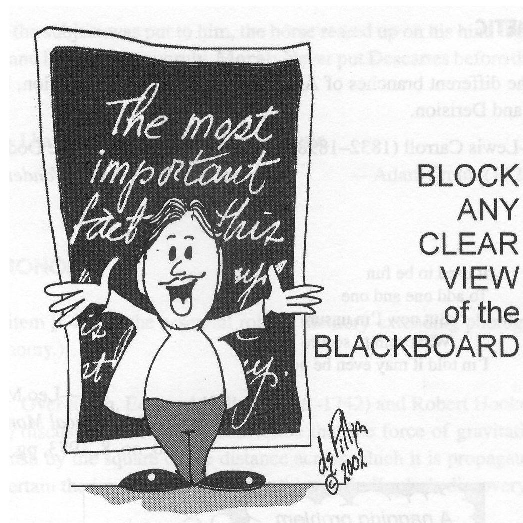
The MA Comprehensive and PhD Preliminary Examinations in Statistics are based on the material in a standard course in Mathematical Statistics at the undergraduate level. Students taking this exam are assumed to have a fairly complete mastery of this material. The lists below contain topics for the exam and titles of a number of standard mathematical statistics textbooks that could be used to prepare for these exams. Any one of these books would be quite satisfactory. The material on the examination can also be found in the course Math 441–2. *The exams will also have a take home computer component where you will be asked to run a statistical analysis on some data using at least one piece of statistical software.*

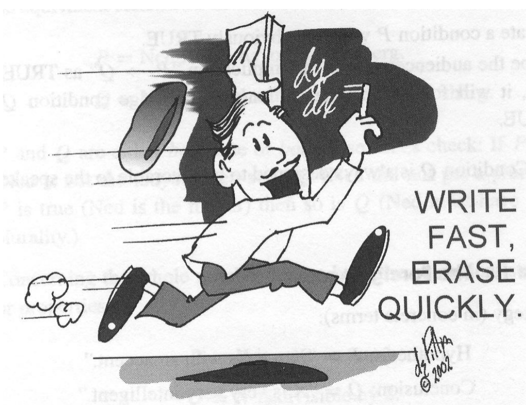
Possible topics for the Statistics examinations:

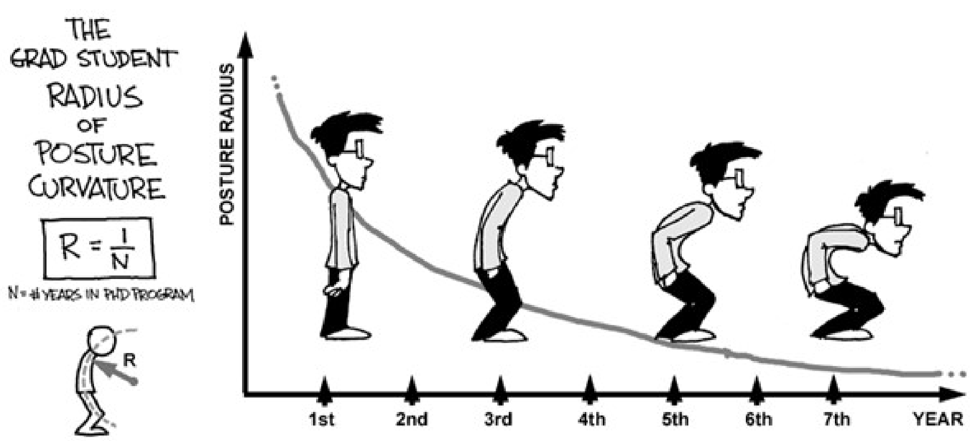
1. Probability
   (a) Sample spaces, events, probability axioms
   (b) Counting techniques
   (c) Conditional probability, independent events
   (d) Discrete and continuous sample spaces

2. Random Variables
    (a) Probability distributions for random variables
    (b) Distribution functions and density functions
    (c) Expected values, moments and generating functions
    (d) Functions of random variables
3. Particular discrete and continuous probability models
    (a) Bernoulli and Binomial
    (b) Geometric and Negative Binomial
    (c) Hypergeometric
    (d) Poisson
    (e) Uniform, Exponential and Gamma
    (f) Beta and Normal
4. Jointly distributed random variables
    (a) Vector random variables
    (b) Conditional distributions and independence
    (c) Expected values, moments, sums of random variables
    (d) Chebyshev’s inequality and the law of large numbers
    (e) The central limit theorem
    (f) Multinomial, bivariate normal, t, F, chi-square distributions
    (g) Order statistics
5. Methods for deriving probability distributions for random variables
    (a) Distribution function technique
    (b) Change of variable technique
    (c) Moment generating function technique
6. Descriptive statistics and graphical displays
    (a) Sample mean and variance, median, percentiles, range, interquartile range
    (b) Histograms, stem and leaf plots, boxplots, probability or quantile plots
7. Estimation of parameters
    (a) Methods of moments and maximum likelihood
    (b) Properties of estimators

(c) Confidence interval estimation







8 Graduate Courses for the Academic Year

The following table contains the graduate courses and seminars scheduled to be offered during Fall semester; it also lists 400-level undergraduate courses that graduate students may take for graduate credit (UG courses).

8.1 Fall Semester 2023

CRN	Course	Title	Days	Time	Room	Instructor
UG-Courses						
70950	STAT 421	Probability Theory	MWF	12:00-12:50	108	Graham, Jonathan
70062	M 431	Abstract Algebra I	MTWF	1:00-1:50	311	Vonessen, Nikolaus
70124	M 439	Euclidean & Non-Euclidean Geo	MWF	11:00-11:50	306	Sriraman, Bharath
74301	M 473	Introduction to Real Analysis	MTWF	11:00-11:50	311	Gillaspy, Elizabeth
72795	M 485	Graph Theory	TR	9:30-10:50	211	Kayll, Mark
Graduate Courses						
74302	M 521	Advanced Algebra I	MWF	9:00-9:50	305	Vonessen, Nikolaus
73415	M 540	Numerical Methods	MWF	2:00-2:50	306	Perez Alvaro, Javier
74304	M 555	Functional Analysis	MWF	2:00-2:50	312	Stroethoff, Karel
72025	M 561	Advanced Analytics	MWF	9:00-9:50	306	Perez Alvaro, Javier
74305	M 581	Combinatorics	MWF	10:00-10:50	211	
74306	M 609	Math Ed Research Methods				Wu, Ke
Seminars						
72592	M 504	Topics in Math Education	T	11:00-11:50	108	Roscoe, Matthew
70063	M 600	Colloquium	M	3:00-3:50	103	
70114	M 610	Gr Sem in Applied Math	T	3:00-3:50	211	Kalachev, Leonid
71823	M 650	Gr Sem in Analysis	W	4:00-4:50	311	
70119	M 694	College Teaching Seminar				

8.2 Spring Semester 2024

The listings for Spring semester are tentative. For more information regarding these courses, consult course announcements, catalog descriptions, or the instructor.

Course	Title	Room	Days	Time	Instructor
UG Courses					
STAT 422	Mathematical Statistics	Math 211	MWF	10:00-10:50	Jon Graham
M 429	History of Mathematics	Math 211	MWF	12:00-12:50	Matt Roscoe
M 472	Intro to Complex Analysis	Math 306	MTWF	11:00-11:50	Karel Stroethoff
Graduate Courses					
M 522	Advanced Algebra II	Math 211	MWF	1:00-1:50	Nikolaus Vonessen
M 562	Advanced Theor. Data Analytics	Math 311	MWF	2:00-2:50	Perez Alvaro, Javier
M 567	Advanced Data Science Projects	Math 306	MWF	11:00-11:50	Perez Alvaro, Javier
M 584	Topics in C & O	Math 211	TTh	9:30-10:50	Mark Kayll
M 605	Learning Theories in Mathematics	TBD			Ke Wu
Seminars					
EDU 694	Topics in Math Education	Math 108	T	11:00-11:50	Georgia Cobbs
M 600	Math Colloquium	Math 103	M	3:00-3:50	Cory Palmer
M 610	Gr.Sem. in Applied Mathematics	Math 211	T	3:00-3:50	
M 650	Gr Sem in Analysis	Math 211	W	4:00-4:50	
M 680	Sem: Combinatorics & Optim.	Math 211	W	3:00-3:50	

"I'm a mathematician. My problems are both real and complex."

9 Transition from Master's to PhD Program

Acceptance into the PhD program after completing a master's degree in our program is not automatic: an application needs to be submitted, and some of the application materials that were used to gain admission into the master's degree program should be updated. The Graduate School requires an on-line application form. In addition, the Graduate Committee requires an updated statement of purpose, unofficial transcript and three reference letters from faculty from our program. The Graduate Committee will also want to see how a student's proposed doctoral program fits in with the work done as part of the student's master's degree program. Master's degree students who have entered with deficient background for a PhD degree in our program should take course work that will remedy these deficiencies before entering the PhD program.

10 Research credits for a PhD program

A total of 60 credits is required for a PhD. The appropriate course number for dissertation research is Math 699. Since Math 699 is repeatable for up to 9 credits (see Catalog), no more than 9 credits of Math 699 can be counted toward the degree. If more research credits are desired, they can be taken under Math 597 (which is repeatable up to 12 credits). No more than a combined total of 21 dissertation and research credits may be applied toward the PhD degree (see Graduate Guide-PhD1).

11 Program of Studies Forms

The student and the student's advisor design a program of study for each student. Each year the student must complete (or update) an advisor-approved Program of Study form which is to be kept on file in the Mathematics office. A revised form must be filed if there are any changes to the student's program during the year. This can be done by editing, initializing and resubmitting the approved form on file.

Please note the following deadlines to submit completed (or updated) Program of Studies forms (submit via moodle):

Fall	August 30
Spring	January 18

Program of Studies forms are available at https://www.umt.edu/math/graduate/current-grad.php.

Please avoid the following common mistakes:

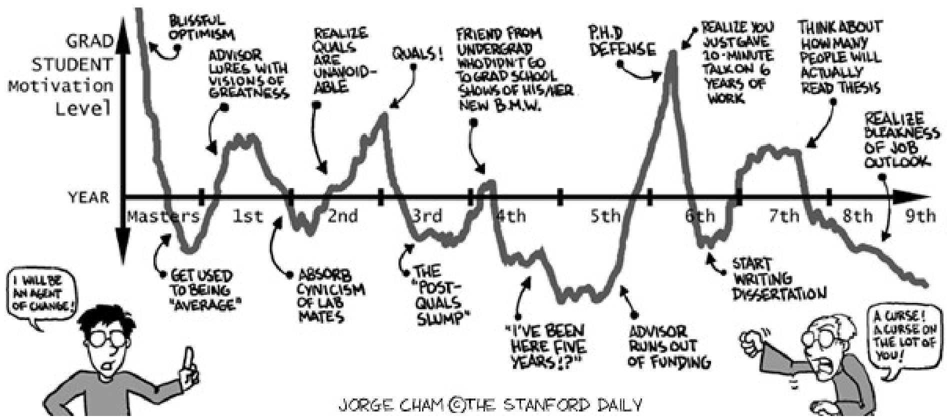
- Program of studies forms submitted without signature of an advisor. Since a plan of studies is to be devised in consultation with an advisor, programs of studies are to be advisor approved. Forms without an advisor's signature will not be accepted.
- Not all undergraduate courses can be used toward the graduate student's credit load; only undergraduate courses labeled UG in the catalog may be used. If a graduate student wants to learn an undergraduate subject by taking 100, 200 or 300 level courses in this subject (for example in a foreign language or in computer science), there is no objection to this if the total number of graduate or UG courses is at least that specified by a student's

program.

- Please submit your program of studies form by the deadline. The Graduate Committee reviews these forms prior to the end of the spring semester for continuing students and the beginning of fall semester for new students so that corrections can be made, if necessary. It is important that the program of studies forms are submitted on time.

12 MA Research Component

Every MA degree program must contain a research project in which the student writes a Master's Thesis, Professional Paper or does an In-house Project. At the start of the student's research the student should form an Examining Committee comprised of a minimum of three faculty members.



The Examining Committee will then be approved by the Graduate Committee. In the case of a master's thesis or professional paper the Examining Committee must also be approved by the Graduate School. The size and composition of this committee is specified by Graduate School Policy C6.000, which reads:

C6.100 — Examining Committee Composition—MA Thesis/Professional Paper

The examining committee shall be comprised of a minimum of three voting members as follows:

1. A qualified UM faculty member or adjunct from the program or unit granting the degree who shall serve as chair;
2. A second qualified UM faculty member or adjunct from the program or unit granting the degree, or from a cooperating program or unit in the case of interdisciplinary degree programs;
3. A qualified UM faculty member or adjunct from a program or unit other than the one granting the degree whose primary responsibility is to ensure that the student is held to reasonable academic standards, that the student is treated fairly by all committee members, and that the student's progress is not unduly delayed by failure of committee members to act in a timely manner.

Note that the above policy requires one member on the committee to be from outside our department; the Graduate School will not approve of the committee if it does not have a member from outside the department.

There are three ways to complete the research requirement for the master's degree:

(i) Master's Thesis. The Examining Committee must be approved by the Graduate School and the Graduate Committee; note that the committee must contain a member from outside the Department of Mathematical Sciences.

(ii) Professional Paper for the Library. The Examining Committee must be approved by the Graduate School and the Graduate Committee; note that the committee must contain a member from outside the Department of Mathematical Sciences.

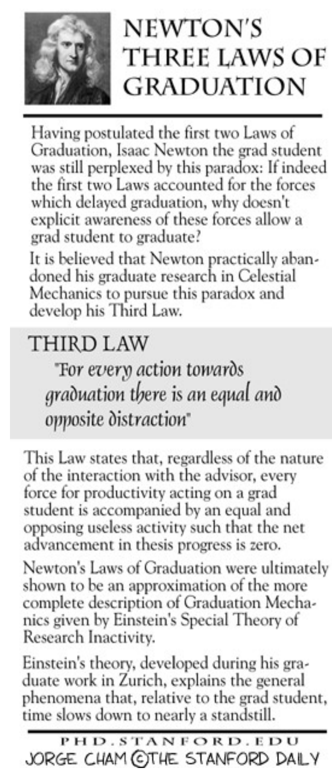
(iii) In-house Project. The Examining Committee requires only Graduate Committee approval; note that the committee need not have a member from outside the Department of Mathematical Sciences.

To form your Examining Committee, first consult with your adviser regarding faculty that may be appropriate to serve based on their interests, qualifications, and availability. Contact each faculty member to determine his or her availability and willingness to serve. Then request approval from Graduate Chair by submitting a Committee Request Form, available on-line. Upon approval by Graduate Committee, the Examining Committee will be recommended to the Graduate School for appointment.

13 PhD Comprehensive Exams and Beyond

13.1 Comprehensive Examination

Following the PhD Preliminary Exams, a doctoral student needs to select a Comprehensive Examination Committee, consisting of the student's advisor and at least three additional members of the mathematics faculty and a faculty member from a cognate field. To form this committee, contact each faculty member to determine his or her availability and willingness to serve. Then request that the Graduate Chair recommend the committee for appointment to the Graduate School. The Comprehensive Examination Committee's duty will be to design, administer and assess a written Comprehensive Examination. A syllabus of the topics to be covered on this examination will be prepared by the Comprehensive Examination Committee in a meeting in consultation with the student. The examination emphasizes, but is not restricted to, the area of specialization of the student. The specific areas and form of the examination are to be determined by the examining committee. The Graduate Committee must approve of the syllabus and form of the examination at least one month prior to the examination date.



A LaTEX template for a PhD Comprehensive Examination proposal is available on-line.

13.2 Advancement to Candidacy

A doctoral student is “advanced to candidacy” when the student has satisfied the Preliminary Examination requirement (see section PhD 3 in the “Guide to Graduate Programs in Mathematical Sciences”) and Comprehensive Examination requirement (see section PhD 5 in the “Guide to Graduate Programs in Mathematical Sciences”).

13.3 Dissertation and Dissertation Proposal

Once an advisor accepts a doctoral student as a dissertation advisee, the student needs to select a Dissertation Committee, consisting of the student’s advisor and four other faculty members. At least one faculty member must be, and two may be, from outside the Mathematics department. To form this committee, contact each faculty member to determine his or her availability and willingness to serve. Then submit a Committee Request Form the Chair of the Graduate Program.

Doctoral students should submit a proposal and research schedule to the committee before the beginning of the student's dissertation research. This proposal is for your use and your committee's guidance and in no sense is a contract.

14 Writing of MA Project/PhD Dissertation

The books by Gillman [5] and Krantz [6] discuss various aspects of mathematical writing[1].

The typesetting system TEX designed by Donald Knuth in the late 1970s, and LATEX, a collection of high-level macros built on TEX developed by Leslie Lamport in the early 1980s, have become the standard for typesetting mathematical documents. The following book provides an introduction to LATEX that goes well beyond the basics.

> Helmut Kopka and Patrick W. Daly, *A Guide to LATEX*, fourth edition, Addison Wesley, Boston, 2004.

A more comprehensive treatment of LATEX is contained in the new edition of the *LATEX Companion* (which is over 1000 pages):

> Frank Mittelbach and Michel Goossens, with Johannes Braams, David Carlisle, and Chris Rowley, *The LATEX Companion*, second edition, Addison Wesley, Boston, 2004.

1. A list of references is given on page 28.

15 Teaching Assistants

A teaching assistant normally has duties equivalent to teaching 3 to 4 hours per week. First time teaching assistants are required to enroll in the College Teaching Seminar Math 694.

The following book

> Steven G. Krantz, *How to Teach Mathematics,* second edition, American Mathematical Society, Providence, 1999,

contains advice on a large number of issues that relate to teaching at the post-secondary level; views on teaching different than those of Krantz are included in more than a dozen appendices.

Bruce Reznick has posted a version of his booklet for teaching assistants (originally published by Random House Birkhauser in 1985) online: http://www.math.uiuc.edu/~reznick/ciu.html

> Bruce Reznick (University of Illinois at Urbana-Champaign), *Chalking It Up: Advise to A New TA,* third edition, 1999.
>
> URL: http://www.math.uiuc.edu/~reznick/ciu.html

Tom Rishel is maintaining an online handbook for teaching assistants:

> Tom Rishel (Cornell University & The Mathematical Association of America), *A Handbook for Mathematics Teaching Assistants,*
>
> URL: http://www.maa.org/programs/tahandbook.html

16 Colloquium talk requirement

According to PhD 1 d) 3 an Option II student must do a current topic project which requires the student to present an expository talk in the Mathematics Colloquium series. The Guide contains the following language about the subject of this colloquium presentation:

> The subject of the talk, to be agreed upon by the student and their advisor, should be based on reading in the current mathematics literature, or on a study of mathematical literature, or on the student’s experiences in an applied consulting project.

Since the Option I student is to give a Colloquium talk on the student’s dissertation (see PhD 8, which requires an expository talk in a mathematics colloquium in the general area of the dissertation topic, for both options), the above mentioned requirement means that an Option II student is to present an additional Colloquium on a current topic. Note that this colloquium talk is in addition to a colloquium talk on the student’s dissertation as required in PhD 8.

17 Example of time line for MA degree

		Autumn Semester			Spring Semester
Year 1		Coursework: 2–3 courses Teaching Seminar		Comp Exam	Coursework: 2–3 courses Colloquium
Year 2	Comp passed	Coursework: 2 courses Seminar Research	Comp passed	(last chance)	Coursework: 2 courses Seminar Research

Remarks

- Research should be taken under M 597 as well as M 593 and 599.

18 PhD Plan

18.1 PhD Plan direct from BA (Additional Option II req in parentheses)

		Autumn Semester			Spring Semester	
Year 1		Coursework: 2–3 courses Teaching Seminar		Transfer Courses	Coursework: 2–3 courses Colloquium	
Year 2		Coursework: 2 courses Seminar		Prelim 1 passed	Coursework: 2 courses Seminar (Prep Teach Internship)	
Year 3	Prelim 2 passed	Coursework: course(s) Seminar Research (Teaching Internship)	Comp Exam Prop	Dissertation Committee	Coursework: course(s) Seminar Research (Current Topic Project)	Comp Passed Adv to Candidacy
Year 4		Coursework: course(s) Seminar Research	(Dissertation Proposal)		Coursework: course(s) Seminar Research	(Coll Talk Current Topic)
Year 5		Research	(Coll Talk Current Topic)		Research PhD completed	Diss Defense

18.2 PhD Plan including an MA (Additional Option II req in parentheses)

		Autumn Semester			Spring Semester	
Year 1		Coursework: 2–3 courses Teaching		Comp Exam	Coursework: 2–3 courses Colloquium	
Year 2	Comp passed	Coursework: 2 courses Seminar Research			Coursework: 2 courses Seminar Research	

		Autumn Semester			Spring Semester	
Year 1		Coursework: 2–3 courses Teaching Seminar		Transfer Courses	Coursework: 2–3 courses Colloquium	
Year 2	Prelim 1	Coursework: 2 courses Seminar		Prelim 2 passed	Coursework: 2 courses Seminar (Prep Teach Internship)	Comp Exam Prop
Year 3		Coursework: course(s) Seminar Research (Teaching Internship)	Comp Passed Adv to Candidacy	Diss Committee	Coursework: course(s) Seminar Research (Current Topic Project)	(Diss Proposal)
Year 4		Coursework: course(s) Seminar Research	(Coll Talk Current Topic)		Coursework: course(s) Seminar Research PhD completed	Coll Talk Diss Diss Defense

Remarks

- The number of courses per semester will depend on the student's preparation prior to entering the program. It is strongly recommended that doctoral students submit a request for transfer of graduate course work completed prior to admission into our program or before the start of their second semester in the program. Obviously, the amount of coursework in subsequent semesters will be dependent upon approval of transferred coursework, so to plan for timely completion of the course requirements it is important that transfer requests be made as early as possible (which, according to Graduate School policy D2.101 is after one semester in the program).

- The indicated times for passing the two preliminary exams in the above chart are the latest these exams can be passed according to our Guide.
- Doctoral students in option II need to plan a Teaching Internship one semester prior to doing this Teaching Internship. The appropriate course number for the Teaching Internship and/or its preparation is M 690. Of course, this Teaching Internship can be done any semester; in the chart we have put it between completion of the Preliminary and Comprehensive Examination requirements.
- The chart shows the study of the current topic project (appropriate course numbers are M 593 and M 597, since this current topic project is separate from the dissertation research) in the student's 6th semester of the program. The colloquium talk based on this current topic project is best presented prior to the end of the 7th semester in the program.

Depending on the number of completed credits, research should be taken under M 597 as well as M 699. The appropriate course number for dissertation research is M 699. However, since M 699 is repeatable for up to 9 credits, no more than 9 credits of M 699 can be counted toward the degree. If more research credits are desired, then they can be taken under M 597 (which is repeatable up to 12 credits).

19 Graduation Form

The graduation application is available on-line. (The graduation application is on the Graduate School website.) Please submit to the graduate chair for approval. Upon approval, the graduate application form will be submitted to the graduate school. The application is due at the end of the semester prior to the semester of graduation. For this academic year the following deadlines apply:

1st Monday in February	Spring Graduation Applications for MA and PhD due. All requirements must be completed by June 19. Please submit paperwork to graduate chair two weeks prior to this deadline.
May 12	Last day of Spring Semester and award date for Spring degrees
1st Monday in June	Summer Graduation Applications for MA and PhD due All requirements must be completed by August 31.
July 31	Last day of Summer Semester and award date for Summer degrees
1st Monday in October	Fall Graduation Applications for MA and PhD due

20 Resources

Following are several useful resources for graduate students in the mathematical sciences. Krantz's Survival Guide contains advice from preparing and applying to graduate school to the life of an assistant professor.

Most of the cartoons used in this "Supplement" come from Jorge Cham's website *Piled Higher and Deeper* [11], which contains hundreds of cartoons depicting graduate life (or lack thereof). The cartoons on pages 10, 16, 20 and 24 (top two) come from John dePillis' book 777 Mathematical Conversation Starters [12], which in addition to illustrations by the author contains numerous quotations from mathematicians, as well as mathematical poems and songs.

References

Teaching

[1] Steven G. Krantz, *A Mathematician's Survival Guide: Graduate School and Early Career Development,* American Mathematical Society, Providence, RI, 2003.

[2] Steven G. Krantz, *How to Teach Mathematics,* second edition, American Mathematical Society, Providence, RI, 1999.

[3] Bruce Reznick, *Chalking It Up: Advise to A New TA,* third edition, 1999. URL: http://www.math.uiuc.edu/~reznick/ciu.html

[4] Tom Rishel, *A Handbook for Mathematics Teaching Assistants,* http://www.maa.org/programs/tahandbook.html

Writing

[5] Leonard Gillman, *Writing Mathematics Well: A Manual for Authors,* Mathematical Association for America, Washington, DC, 1987.

[6] Steven G. Krantz, *A Primer of Mathematical Writing,* American Mathematical Society, Providence, RI, 1997.

Typeset

[7] Donald E. Knuth, *The TEXbook,* Addison Wesley, Reading, MA, 1984.

[8] Helmut Kopka and Patrick W. Daly, *A Guide to LaTeX,* fourth edition, Addison Wesley, Boston, MA, 2004.

[9] Lesley Lamport, *LaTeX: A Document Preparation System,* second edition, Addison Wesley, Reading, MA, 1994.

[10] Frank Mittelbach and Michel Goossens, with Johannes Braams, David Carlisle, and Chris Rowley, *The LaTeX Companion*, second edition, Addison Wesley, Boston, MA, 2004.

Humor

[11] Jorge Cham, *Piled Higher and Deeper: A Graduate Student Comic Strip,* http://www.phdcomics.com

[12] John dePillis, *777 Mathematical Conversation Starters,* The Mathematical Association of America, Washington, DC, 2002.